Expert Systems With Applications 233 (2023) 120901

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

A comprehensive review of visualization methods for association rule mining: Taxonomy, challenges, open problems and future ideas

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ARTICLE INFO

Keywords:
Association rule mining
Numerical association rule mining
Data mining
Visualization
Plots

ABSTRACT

Association rule mining is intended for searching for the relationships between attributes in transaction databases. The whole process of rule discovery is very complex, and involves pre-processing techniques, a rule mining step, and post-processing, in which visualization is carried out. Visualization of discovered association rules is an essential step within the whole association rule mining pipeline, to enhance the understanding of users on the results of rule mining. Several association rule mining and visualization methods have been developed during the past decades. This review paper aims to create a literature review, identify the main techniques published in peer-reviewed literature, examine each method's main features, and present the main applications in the field. Defining the future steps of this research area is another goal of this review paper.

1. Introduction

Association Rule Mining (ARM) is definitely one of the most important and popular data mining techniques for discovering unknown knowledge from transaction databases. The ARM is also a part of Machine Learning (ML) with the task to discover interesting relationships between items in large transaction datasets. The relationships are expressed by association rules determining how and why certain items are connected. The story of ARM started with a seminal paper of Agrawal, Srikant, et al. (1994). Agrawal set the theoretical foundations for the process of ARM, and proposed the first algorithm, called *Apriori*. Apriori is a deterministic algorithm for mining association rules, and is still today featured as one of the top algorithms in the Data Mining (DM) domain (Wu et al., 2008), as well as a member of an unprecedented scale in student textbooks.

Many of the first ARM algorithms were based on deterministic methods like Eclat (Zaki, 2000) and FP-growth (Borgelt, 2005; Han, Pei, & Yin, 2000). However, contemporary ARM algorithms are based on stochastic population-based methods (Telikani, Gandomi, & Shahbahrami, 2020). A substantial effect on this domain was contributed by Altay and Alatas (2019), who utilized these methods for dealing with both categorical and numerical attributes. On the other hand, the ARM algorithms have started to apply for mining sequential patterns. For instance, Soysal (2015) replaced the traditional find-all-then-prune approach with the heuristic method to extract primarily associated patterns without pruning.

In the following years, the ARM gained huge interest in the ML community. Its popularity was proven with many practical applications, especially, in the domains such as market-based analysis (Nisbet, Miner, & Yale, 2018), medical diagnosis (Xu, Zhao, Zhan, Wang, & Hu, 2022), census data (Malerba, Lisi, Appice, & Sblendorio, 2002) or protein sequences (Gupta, Mangal, Tiwari, & Mitra, 2006), among others.

Data analysis pipelines typically consist of data cleanup and minimizing data imputations (also data pre-processing), data collection and exploration design, and comprehending the mined knowledge. Thus, the whole ARM pipeline is complex (see Fig. 1), because it consists of three steps, as follows: the pre-processing, the ARM, and the post-processing. The input to the pipeline presents the transaction database, which consists of rows and columns, where each row presents a transaction, and columns the attributes. In the pre-processing step, some optional substeps can be applied to make the data more robust, i.e., data cleaning and missing data imputation, where some outliers, or rows with a lot of missing data can even be removed. On the other hand, some other operations, for example, data squashing (Fister, Fister, Novak, & Verber, 2022), can help reduce the transaction dataset. Then, the ARM process itself is performed. In line with this, several

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https://doi.org/10.1016/j.eswa.2023.120901
Received 6 March 2023; Received in revised form 5 June 2023; Accepted 21 June 2023
Available online 27 June 2023

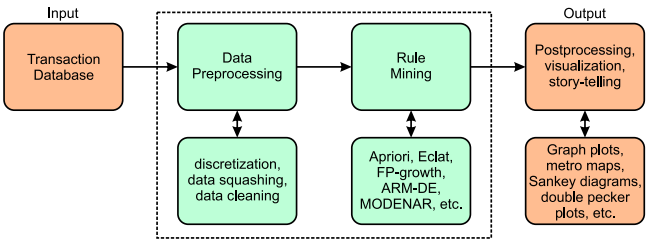


Fig. 1. The basic ARM pipeline.

algorithms exist, as mentioned, and are some of the most used. The output of this step is usually a huge collection of mined/identified/found association rules. Usually, researchers present these rules as a table, or summarize them using some metrics. However, visualization of the association rules needs to be conducted for the best insights.

Nowadays in ML, there is a trend to go for easier representation of the results obtained by ML/AutoML (automated ML) pipelines. This intention also coincides with the emerging research area of eXplainable Artificial Intelligence (XAI) (Arrieta et al., 2020; Barredo Arrieta et al., 2019). XAI has become an important part of the future of AI, because XAI models explain the reasoning behind their decisions. This provides an increased level of understanding between humans and machines, which can help build trust in AI systems (Kumar, 2022). In summary, XAI is a set of processes and methods to comprehend and trust the results created by the ML algorithms. In line with this, it tries to describe an AI model's impact on the one hand, and exposes its potential biases on the other. Thus, the ML model is estimated according to its accuracy, fairness, transparency, and outcomes of AI-powered decision-making (Borrego-Díaz & Galán-Páez, 2022).

XAI can be manifested in several forms: text explanation, visualization, local explanation, explanation by example, explanation by simplification, and feature relevance (Barredo Arrieta et al., 2019; Bennetot et al., 0000). Thus, there is an increased interest of researchers in developing new methods for easier representation of the results. Definitely, one of the most important parts of these efforts is visualization methods (Arrieta et al., 2020).

Typically, ARM algorithms generate a huge number of association rules. Frequently, the results are opaque for ordinary users, and need some explanations to understand their meaning. On the other hand, visualization of the results has a huge explanation power. Although a lot of visual methods have been proposed for ARM, to the best of our knowledge, no review for dealing with this problem from the XAI point of view exists nowadays.

Therefore, the aim of this paper is to collect and discuss visualization techniques for ARM that have appeared from its advent to the present day. Each method is studied in detail and features are compared with each other in the sense of XAI. The contributions of this review paper are summarized as follows:

- The evolution of ARM visualization methods is presented.
- The features of each of the methods are defined.
- The advantages/disadvantages of each method are outlined.
- An example is presented for each of the surveyed methods.
- Explaining models using the ARM visualization are summarized.

The review of the ARM visualization methods is based on papers published from three different main sources: the ACM Digital Library, IEEEXplore, and Google Scholar. The analyze of the methods are highlighted from the following points of view: (1) characteristics, (2) visualization focus, and (3) attribute type. The taxonomies of the ARM visualization methods are introduced based on the highlights.

The structure of the paper is organized as follows: Section 2 deals with the ARM problem in a nutshell. The mathematical definition of the ARM visualization is the subject of Section 3. A detailed overview of traditional ARM visualization methods is reviewed in Section 4. New ideas in the ARM visualization are the subject of Section 5. Taxonomies of the ARM visualization are illustrated in Section 8. In Section 9, review of the remainder of the analyzed ARM visualization paper follows. The review concludes with Section 10 that summarizes the performed work and outlines potential ideas for the future work.

2. Association rule mining in a nutshell

The ARM problem is defined formally as follows: Let us suppose a set of items $I = \{i_1, \ldots, i_M\}$ and transaction database $D = \{Tr_1, \ldots, Tr_N\}$ are given, where each transaction Tr_i is a subset of objects $Tr_i \subseteq I$. Thus, the variable M designates the number of items, and N the number of transactions in the database. Then, an association rule can be defined as an implication:

$$X \Rightarrow Y, \tag{1}$$

where $X \subset I$ (left-hand-side or LHS), $Y \subset I$ (right-hand-side or RHS), and $X \cap Y = \emptyset$. The following four measures are defined for evaluating the quality of the association rule (Agrawal et al., 1994):

$$supp(X \Rightarrow Y) = \frac{n(X \cap Y)}{N}, \tag{2}$$

$$conf(X \Rightarrow Y) = \frac{n(X \cap Y)}{n(X)}, \tag{3}$$

$$lift(X \Rightarrow Y) = \frac{supp(X \cap Y)}{supp(X) \times supp(Y)}, \tag{4}$$

$$conv(X \Rightarrow Y) = \frac{1 - supp(Y)}{1 - conf(X \Rightarrow Y)}, \tag{5}$$

where $supp(X \Rightarrow Y) \geq S_{min}$ denotes the support, $conf(X \Rightarrow Y) \geq C_{min}$ the confidence, $lift(X \Rightarrow Y)$ the lift, and $conv(X \Rightarrow Y)$ the conviction of the association rule $X \Rightarrow Y$. There, N in Eq. (2) represents the number of transactions in the transaction database D, and $n(.)$ is the number of repetitions of the particular rule $X \Rightarrow Y$ within D. Additionally, S_{min} denotes minimum support and C_{min} minimum confidence, determining that only those association rules with confidence and support higher than C_{min} and S_{min} are taken into consideration, respectively.

The interpretations of the measures are as follows: The support measures the proportion of transactions in the database which contain the items X and Y. The confidence estimates the conditional probability $P(Y|X)$, denoting the probability to find the Y of the rule in transaction under the condition that this transaction also contains the X. The lift is the ratio of the observed support that X and Y arose together in the transaction if both set of items are independent. The conviction evaluates the frequency with which the rule makes an incorrect prediction.

3. Visualization of association rule mining

Visualization of ARM can be described mathematically as a set of triplets:

$$\mathcal{R} = \{\langle X_1, Y_1, Z_1\rangle, \ldots, \langle X_i, Y_i, Z_i\rangle, \ldots, \langle X_n, Y_n, Z_n\rangle\}, \tag{6}$$

where X_i denotes an antecedent, Y_i a consequent, and Z_i a vector of available interestingness measures (e.g., support, confidence, etc.) for $i = 1, \ldots, N$. In a nutshell, different visualization methods depend on:

- the number of interestingness measures to display,
- the visualization focus,
- the rule set size.

The number of interestingness measures to display is limited by the number of dimensions that can be visualized (i.e., 2D or 3D). The visualization focus determines how the association rule defines the neighborhood of rules to be visualized. In line with this, the neighborhood is defined by: interestingness measure, items, similarity of RHS and LHS, or time series' visualization. The rule set size limits the number of association rules that are included into a specific visualization method.

Table 1
Search results of papers regarding the keywords in various databases.

Database name	URL	Total	Included
ACM Digital Library	dl.acm.org	6	4
IEEEXplore	ieeexplore.ieee.org	214	21
Google Scholar	scholar.google.com	16,100	25+
Total		16,320	25+

3.1. Study design

For conducting the systematic literature review, we followed the guidelines presented in the Systematic Literature Review Guidelines in Software Engineering (Kitchenham, Charters, et al., 2007). Our primary goal was to identify the frequency of the ARM visualization methods, the main features of these methods, and the applications in which these methods were applied. According to our goals, we developed the following Research Questions (RQ)s:

- RQ1: Which methods are developed for the ARM visualization?
- RQ2: Which challenges and open problems are placed behind the ARM visualization?
- RQ3: Which software packages are available to users tackling these problems?
- RQ4: What awaits the methods for visualization of association rules in the future?

We conducted a literature search using major databases from 18 to 22 November, 2022. The main search strings that were used for searching the databases were as follows: "association rule mining" AND "visualization" OR "visualisation". The search string was also modified according to the search formats of different databases. The study focused on three databases: Google Scholar, ACM Digital Library, and IEEE Xplore. Google Scholar is the broadest database that includes most of the other database's papers as well. Indeed, this fact was justified when we include the other databases, like ScienceDirect, Scopus, and WOS, for the test. Unfortunately, this inclusion did not improve the search capability by using the same search keywords in the sense of the increased number of papers. On the other hand, the other databases, i.e., ACM and IEEE, were included because these consist of the more prominent papers from the Computer Science domain, where usually all doubts related to the proper title and authors' names of the papers are excluded.

Table 1 presents the results of our search.[1] Each of the papers was prescreened according to its abstract and keywords.

When the results were collected, we also filtered out the duplicates. Additionally, when searching through the Google scholar we checked for citing articles of each paper, so that additional results were then identified and included in this review paper. We also specified the selection and exclusion criteria as well as limitations. The selection criteria were the follows: (1) research paper addresses any kind of ARM and its connection with visualization, and the research must be peer reviewed, i.e., published in a referred conference, journal paper, book chapter or monograph. The search was conducted with exclusion criteria as follows: "The research paper is not written in the English language", and limitations such as: "The literature review search was limited to only three databases".

The summary of abstracts from IEEEXplore and ACM Digital Library publications is shown in the wordcloud Fig. 2, from which it can be seen which keywords mainly highlight the research area studied in this review paper.

The frequency of papers is presented in Fig. 3. Let us mention that only articles that fully utilize/present/propose/use visualization methods are included in a bar chart. According to the figure, we can see that there are no notable deviations during the years, while there are some islands in the specific years when no papers were published in this research area.

[1] Note that we also checked the citing articles of results from Google Scholar manually.

Fig. 2. Wordcloud of the extracted abstracts.

4. A detailed overview of traditional ARM visualization methods

In the following subsections, each of the methods is outlined, followed by a summary of related work, while several methods are also illustrated by an example. The examples of the particular visualization are implemented in arulesViz (Hahsler, Chelluboina, Hornik, & Buchta, 2011) on a set of 11,267 association rules produced by the Apriori algorithm (Agrawal et al., 1994) mining the Mushroom UCI ML dataset (UC Irvine ML Repository, 1987) using the following limitations: $S_{min} = 0.3$ and $C_{min} = 0.5$.

Table 2 presents a summary of the traditional ARM visualization methods that were found in our systematic literature review. It is divided into four columns that present: a sequence number (column 'Nr.'), a class (column 'Class'), a variant (column 'Variant'), and the method's developer (column 'Reference'). As can be seen from the table, we are focused on eight classes of visualization methods and their variants (together seven visualization methods). In the remainder of the paper, the aforementioned visualization methods are illustrated in a nutshell.

4.1. Scatter plot

A Scatter plot (Fig. 4(a)) was firstly used for visualizing mined association rules by Bayardo and Agrawal (1999). In general, this plot is used to display an association or relationship between interestingness measures Z_i (usual support and confidence) that are presented as dots in the Scatter plot. Additionally, the third measure (usual lift) is included as a color key. Thus, rules with similar values of interestingness measures are placed closer to each other, while the correlation can be established between dependent and independent variables. Typically, the so-called regression line is drawn in the Scatter plot, representing the trend of the relationship between two observed variables. This line can also be used as a predictive tool in some circumstances.

4.1.1. Twokey plot

A two-key plot (Fig. 4(b)) is a special kind of Scatter plot that was developed by Unwin et al. (2001), especially, for analyzing association rules. It consists of a two dimensional Scatter plot displaying an association between two measures of interestingness (usually support and confidence), while the third measure is represented by the color of the points (i.e., support/confidence pairs), where the color corresponds

Table 2
Summary of ARM visualization methods.

Nr.	Class	Variant	Reference
1	Scatter	Scatter plot Two key plot	Bayardo and Agrawal (1999) Unwin, Hofmann, and Bernt (2001)
2	Graph	Graph-based	Klemettinen, Mannila, Ronkainen, Toivonen, and Verkamo (1994)
3	Matrix	Matrix-based Grouped matrix-based	Hian-Huat Ong, leong Ong, keong Ng, and peng Lim (2002) Hahsler and Karpienko (2017)
4	Mosaic	Mosaic plot Double decker plot	Hofmann (2008) Hofmann and Wilhelm (2001)

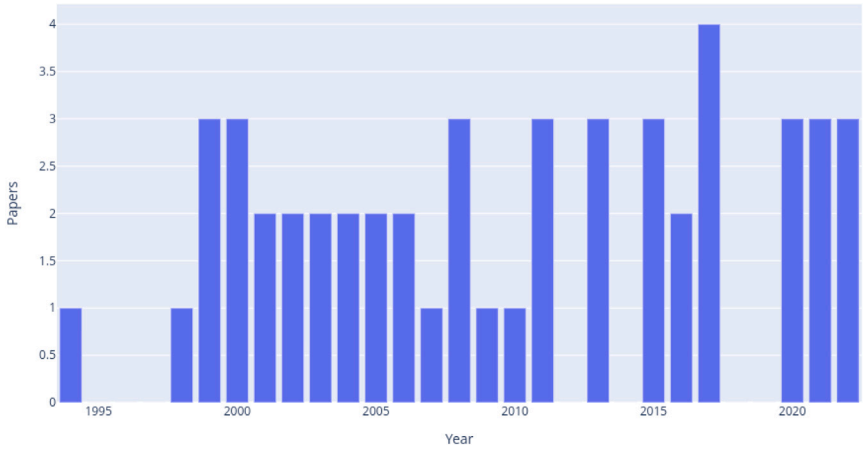
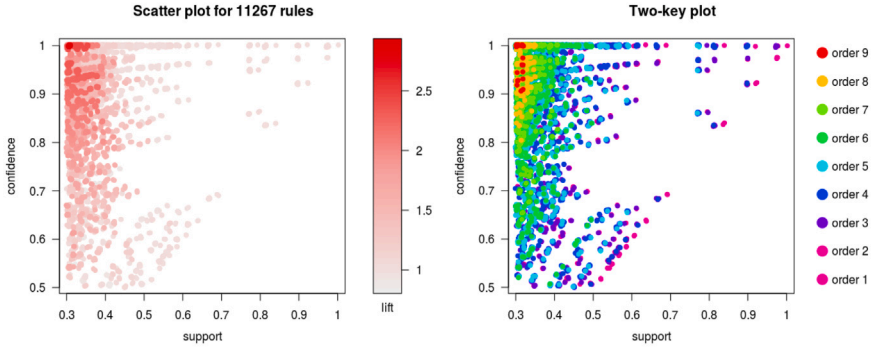


Fig. 3. The frequency of papers per year.



(a) Scatter plot. (b) Two-key plot.

Fig. 4. Scatter and Two-key plots powered by arulesViz.

to the length of the rule (also order). Interestingly, 2-order association rules describe trails moving from the upper right side (perfect result) to the left lower side of the same plot (lesser support and lesser confidence).

4.2. Graph-based

Graph-based techniques (Fig. 5) identify how rules share individual item (Buono & Costabile, 2005; Ertek & Demiriz, 2006; Klemettinen et al., 1994; Rainsford & Roddick, 2000). They visualize association rules using vertices and edges, where vertices annotated with item labels represent items, and itemsets or rules are represented as a second set of vertices. The items are connected with itemsets/rules using arrows. For rules, arrows pointing from items to rule vertices indicate LHS items, and an arrow from a rule to an item indicates the RHS. Interestingness measures are typically added to the plot by using the color or the size of the vertices representing the itemsets/rules. Graph-based visualization offers a very clear representation of rules but they tend to become cluttered easily, and, thus, are only viable for very small sets of rules.

4.3. Matrix-based

Matrix-based visualization (Hian-Huat Ong et al., 2002) (Fig. 6(a)) identifies associations between antecedent (LHS) and consequent (RHS) items. Thus, association rules are organized as a square matrix $M = \{m_{j,k}\}$ of dimension $M \times M$, in which distinct antecedent items $X_i \in \{x_{i,j}\}$ for $j = 1, \ldots, |X_i|$ and distinct consequent items $Y_i \in \{y_{i,k}\}$ for $k = 1, \ldots, |Y_i|$ are included. The values of some interestingness measure (e.g., lift) are then assigned to the corresponding position $m_{j,k} = Z_i$ of the matrix. Typically, the antecedent itemset of the rules is ordered by increasing support, while the consequent itemset by increasing confidence before visualization.

However, the matrix visualization is limited by the rule set size (i.e., <1000), especially in the case of a huge matrix, which makes the exploration of the matrix much harder.

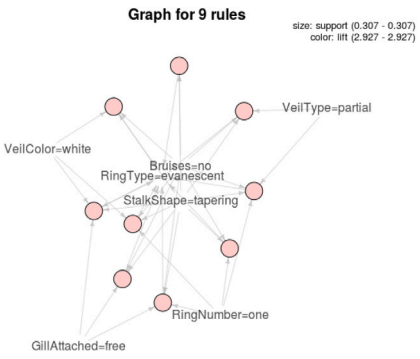


Fig. 5. Graph plot powered by arulesViz.

4.3.1. Grouped matrix-based visualization

The grouped matrix-based visualization (Hahsler & Karpienko, 2017) (Fig. 6(b)) is a variant of the original matrix-based visualization, where the large set of different antecedents (the columns in matrix M) are grouped into the smaller set of groups using clustering. Mathematically, the set of antecedents is grouped into a set of k groups $S = \{S_1, \ldots, S_k\}$ according to minimizing the sum of squares within the particular cluster, in other words:

$$\arg\min_{S} \sum_{i=1}^{k} \sum_{m_{i,j} \in S_i} \|m_{i,j} - \overline{m}_i\|^2, \tag{7}$$

where $\mathbf{m}_i = \{m_{i,j}\}$ for $j = 1, \ldots, |A_i|$ is a column i of matrix $\mathbf{M}$ which represents all values with the same antecedent, and $\overline{m}_i$ is the center of the cluster S_i. Thus, the k-means algorithm (Hartigan & Wong, 1979) is applied 10-times with random initialization of the centroids. The best solution is then used for an ARM visualization. The motivation behind the ARM visualization method is to reduce the antecedent's dimension that enables more informative visualization of the association rules.

4.4. Mosaic plot

A mosaic plot (Hartigan & Kleiner, 1984) is applied for visualizing the interesting rule, consisting primarily of categorical attributes (Fig. 7(a)). It is based on the so-called contingency table, in which the frequencies of the attribute appearances in the interesting rule r^* are assigned to each position $m_{j,k}$, where j denotes the corresponding the antecedent attribute A_j and k the consequent attribute A_k.

The interesting rule is determined as follows: Let us assume that each rule $r_i \in \mathrm{R}$ is a tuple $r_i = \langle X_i, Y_i, Z_i \rangle$, where X denotes the attributes $A = \{A_i, \ldots, A_p\}$ belonging to the antecedent, Y to the consequent, Z is a set of interestingness measures, and $X \cap Y = \emptyset$. Then, the interesting rule r^* for visualizing with mosaic plot is defined as

$$r^* \Rightarrow Y|Z, \tag{8}$$

where $X = \{A_{x_1} = a_{x_1} \wedge A_{x_k} = a_{x_k}\}$, $Y = \{A_y = a_y\}$, and $Z = \{supp, conf\}$, for which the difference of confidence (doc) for rule $X \Rightarrow Y$ and $\neg X \Rightarrow Y$ is the maximum, in other words:

$$\max_{r \in \mathrm{R}} conf(X \Rightarrow Y) - conf(\neg X \Rightarrow Y). \tag{9}$$

Mosaic plots were introduced as a graphical analogy of multivariate contingency tables (Hofmann, Siebes, & Wilhelm, 2000). This means that the position $m_{i,j}$ (also a cell in a contingency table) is presented in a mosaic plot as an area divided into the highlighted part (colored) that is proportional to the support of the rule $X \Rightarrow Y$ and the unhighlighted part of the rule $\neg X \Rightarrow Y$. Thus, the confidence is proportional to the height of the highlighted part of the area.

4.4.1. Double Decker plot

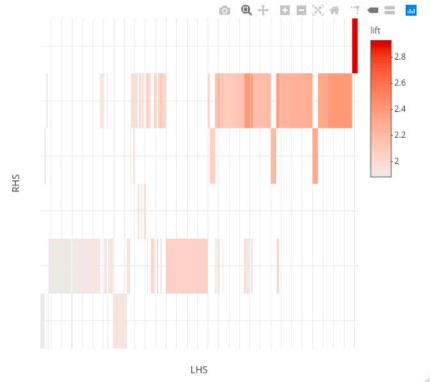
Double Decker plot (Hofmann, 2000) allows comparing the proportions of the highlighted heights referring to confidence measure more easily (Fig. 7(b)). While the original mosaic plot splits tiles in vertical and horizontal directions, the Double Decker splits these only horizontally. As a result, the antecedent of the interesting rule is now expressed mathematically as:

$$X = \{A_{x_1} = \cdot \wedge A_{x_p} = \cdot\},$$

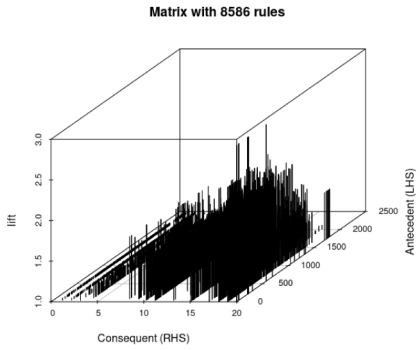
i.e., the proportions of the highlighted heights are presented in each tile of the mosaic plot, while the widths of the tiles are represented as labels denoting the antecedent's attributes. Thus, the highlighted shades illustrate relations with an outcome set to 'True', while the white shades refer to relations, whose outcome leads to 'False'.

5. New ideas in the visualization of association rules

This section reviews papers dealing with ARM visualization methods that accumulate new ideas in this domain. The ideas are collected in Table 3, where the papers are classified according to different visual features. As can be seen from the table, here, we were focused on the seven ARM visualization methods, which, in our opinion, best reflect the development in this domain. In the remainder of the section, the selected ARM visualization methods are illustrated in detail. Let us emphasize that we provide visualization graphics only for those methods which are either publicly available or the permissions to publish were obtained by the corresponding authors.

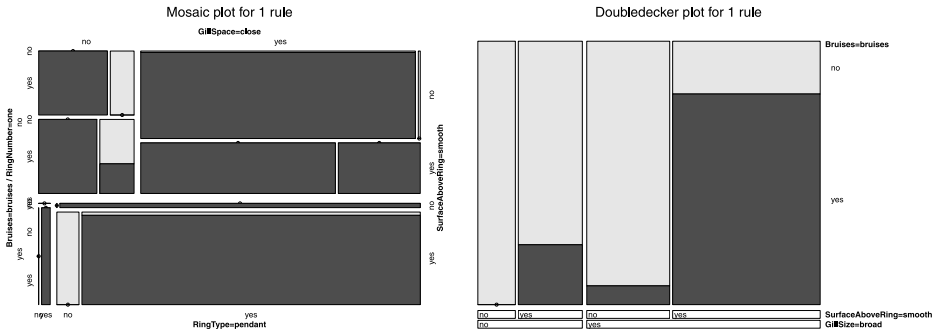


(a) Matrix plot.



(b) Grouped matrix-based plot.

Fig. 6. Matrix and Grouped matrix-based plots powered by arulesViz.



(a) Mosaic plot. (b) Double Decker plot.

Fig. 7. Mosaic and Double Decker plots powered by arulesViz.

Table 3
Summary of the new ARM visualization methods.

Nr.	Class	Variant	Reference
1	Fishbone	Ishikawa diagram	Tsurinov, Shpynov, Lukashina, Likholetova, and Artyomov (2021)
2	Molecular	Molecular representation	Said, Guillet, Richard, Picarougne, and Blanchard (2013)
3	Lattice	Concept lattice	Shen, Bao, and Zhang (2020)
4	Metro	Metro map	Fister and Fister (2022b)
5	Sankey	Sankey diagram	Fister and Fister (2022a)
6	Ribbon	Ribbon plot	Fister et al. (2020)
7	Glyph	Glyph-based	Hrovat, Fister, Yermak, Stiglic, and Fister (2015)

5.1. Ishikawa diagram

Typically, the Ishikawa diagram (Tague, 2005) is applied as a cause analysis tool appropriate for describing the structure of a brainstorming session, in which a development team tries to identify possible reason causing a specific effect. Consequently, the Ishikawa chart is also called a cause/effect diagram. As a result of the brainstorming process, a fishbone diagram is constructed as an arrow with an arc directing to the effect (i.e., a problem statement). Then, the possible causes of the problem need to be identified that are presented as branches originating from the main arrow.

The diagram has also been applied in ARM visualization. For instance, the authors (Tsurinov et al., 2021) have established that ARM algorithms produce a large number of mined association rules in unstructured form. This means that there is no information about which features are more relevant for a user. In this sense, they proposed the Fishbone ARM (FARM) that is able to introduce a hierarchical structure for rules. The structure enables that the priority of features becomes clearly visible.

The fishbone structure presents a basis for visualization with FARM. In this structure, features, inserted as ribs in a symbolic fishbone, are ordered such that the conviction metric values grow from the rear toward the head. Thus, the complexity of the structure increases by adding additional attributes. On the other hand, the statistical significance of the results also needs to be increased. In line with this, cross-validation is employed for evaluating the significance that splits the result dataset into two different portions (i.e., test and validation), and then re-sampled during more iterations.

5.2. Molecular representation

A molecule is a group of two or more atoms connected together with chemical bounds (e.g., covalent, ionic) (Ebbing & Gammon, 2016). Therefore, a molecule representation refers to a connected graph with nodes denoting atoms and edges denoting the chemical bounds between them. The representation inspired (Said et al., 2013) into developing a new ARM visualization method that is devoted for visualizing items arising in the antecedent and consequent of the selected association rule. Thus, two characteristics need to be determined: (1) the contribution of each item to the rule, and (2) the correlation between each pair of antecedents and each pair of consequents from an archive of association rules. The association rules are explored before visualization according to one of the interestingness measures selected by the user, e.g., support, confidence, and lift.

The contribution of an item in the selected association rule $R = X \Rightarrow Y$ is calculated with measuring the Information Gain (IG) defined by (Freitas, 1998):

$$IG(A_i) = Info(R) - Info(R|A_i), \tag{10}$$

where

$$\begin{aligned} Info(R) &= -\sum_{j=1}^{n} P(R_j)\log P(R_j), \text{ and} \\ Info(R|A_i) &= \sum_{k=1}^{m} P(Ai,k)\left(-\sum_{j=1}^{n} P(R_j|A_{i,k})\log P(R_j|A_{i,k})\right). \end{aligned} \tag{11}$$

Thus, it holds that attributes with higher values of IG are good predictors of the selected rule. In contrast, if items with low or negative IG values are encountered, the selected rules are estimated as irrelevant. On the other hand, the lift interestingness measure (Eq. (4)) is applied for determining the correlations between pairs of items in the antecedent and consequent, respectively.

The visualization of molecular representation is typically realized using sphere 3D graphs (also powered by R), where spheres present items and edges of the different distances' connection between them. The calculated characteristics of items into the selected rule are captured in a sphere graph as follows:

- the size of the sphere is proportional to the value of IG,
- the positive value of IG is a plot in a sphere of one color (e.g., blue), while the negative one in a sphere of another color (e.g., white),

- the distance between two spheres is proportional to the measure lift.

However, authors (Said et al., 2013) simplified the visualization of association rules based on a molecular representation by developing a tool for VISual mining and Interactive User-Centered Exploration of Association Rules (IUCEARVis).

In summary, the main weakness of the molecular structure is that it shows the importance of items to rules, and cannot show the distribution of association rules.

5.3. Concept lattice

A concept lattice is a tool for extracting specific information from massive data. It is obtained after a concept analysis that belongs to the domain of applied mathematics (Truong & Tran, 2010). The results of the concept analysis are aggregated in a data structure that is, typically, presented in a Hasse graph. The Hasse graph consists of concepts representing as nodes in a 2-dimensional lattice, and edges expressing the generalization and instantiation of relationships between the concepts (Shen et al., 2020).

Formally, the concept lattice is defined as a triple $L = \langle O, A, B\rangle$, where O denotes a set of objects, A a set of attributes, and B is a binary relationship matrix $B \subseteq O \times A$ denoting that an object $o \in O$ and attribute $a \in A$ are in a relationship, if $(i, a) \in B$. Thus, a node in the concept lattice is defined as a pair $\langle A, B\rangle$, where the former member is also called an extension $A \in O$ (i.e., a collection of objects), and the latter a connotation (i.e., collection of attributes). Indeed, a combination of objects and attributes is needed for a more comprehensive analysis of the association rules.

The task of the ARM visual algorithms based on the context is to display association rules extracted from concept lattice. Thus, the central area of the visualization interface consists of a 2-dimensional lattice, within which the concepts are positioned as points according to their values of support and confidence. Two lines are attached below and above the lattice: The former represents the objects which have arisen in the antecedent, while the latter the same in the consequent of the potential association rule. Indeed, if there is a relationship between particular object and attribute in the relationship matrix $(i, a) \in B$, the object is connected with the node (concept) using an edge.

The advantages of the ARM visualization based on a concept lattice can be summarized as follows: (1) a deeper understanding of association rules at the conceptual level, and (2) analyzing the relationships between concepts more comprehensively. However, the main weakness of the visualization is that this is only appropriate for visualizing the binary values of objects. In order to overcome the problem, Yang (2005) proposed generalized association rules capable of visualizing the frequent rules in an itemset lattice that presents one item in parallel coordinates. In this way, many-to-many rules can be visualized on the one hand, and the large number of rules as selected by the user can be displayed on the other. Obviously, the advantage of the ARM visualization methods is that the user can limit the number of association rules for visualization interactively by specifying the parameters $S_{\min}$ and $C_{\min}$.

5.4. Metro maps

The concept of information maps enables analysis of data having a "geographical look" (Shahaf, Guestrin, & Horvitz, 2012; Shahaf, Guestrin, Horvitz, & Leskovec, 2015). The look can also be prescribed to mined association rules. Therefore, the idea to visualize these in the form of metro maps has become appreciated (Fister & Fister, 2022b). This means, similar as the metro map can help a user to orientate him/herself in the environment, the information map can help them to understand the information hidden in the mined association rules. Thereby, the metro map is divided into more metro lines, consisting of various metro stops. In the information sense, each metro stop represents an attribute, while the metro lines a linear sequence of the attributes (also different association rules). Mutual connections between the metro lines reveal how an attribute in one association rule affects an attribute in the other, and vice versa. Finally, understanding the linear sequences of attributes and connections between them can even tell stories about the specific information domain.

The metro map is defined mathematically as $\mathcal{M} = (G, \Pi)$, where $G = (A, E)$ denotes an attribute graph of vertices $A = \{A_1, \ldots, A_M\}$, representing attributes and edges $E = \{r_i, \ldots, r_n\}$ representing simple rules (i.e., rules with one antecedent attribute and one consequent attribute), together with incident function ψ_G that associates an ordered pair $\psi_G = (X, Y)$ denoting the implication $X \Rightarrow Y$, and Π is a set of metro lines $\pi \in \Pi$ (Fister & Fister, 2022b). The evolutionary algorithm was applied in Fister and Fister (2022b) for constructing a metro map that must obey the following four objectives: (1) maximum path length τ, (2) maximum map size K, (3) high coverage, and (4) high structure quality.

Indeed, the maximum path length refers to the maximum number of metro stops (i.e., attributes) in a linear sequence. The maximum map size limits the number of metro lines. The coverage is proportional to the lift interestingness measure, where we were interested in rules with a lift value > 1, determining the degree to which the probability of occurrence of the antecedent, and this of the consequent are dependent on one another. The structure quality ensures that the linear sequences of the metro stops are coherent in all metro lines.

An example of a metro map obtained by mining the Mushroom dataset, that was constructed using the parameters $\tau = 6$ and $|\mathcal{K}| = 4$, is illustrated in Fig. 8. Let us notice that the figure is divided into two parts, i.e., a diagram and a table. The diagram presents the visualized metro map, while the table the meaning of the metro stops (attributes).

5.5. Sankey diagram

Similar to the metro map, the Sankey diagram is also focused on "geographical data". Additionally, the kind of visualization enables visualization of hierarchical multivariate data. It is represented as a graph consisting of nodes representing attributes and edges representing connectivity by flows across time. In this diagram, the quality of each connection is distinguished by its weight that is proportional to some of the interestingness measures.

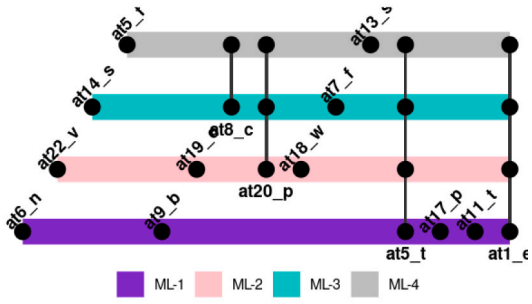
Mathematically, the Sankey diagram is defined as a directed graph $G = \langle K, R\rangle$, where K denotes the maximum path length and R is a set of similar rules (Fister & Fister, 2022a). The rules in this diagram are presented by the antecedent $X = \{A_{x_1} = a_{x_1} \wedge, \ldots, \wedge A_{x_k} = a_{x_k}\}$, representing a set of source nodes, consequent $Y = \{A_y = a_y\}$, representing a set of sink nodes, and interestingness measure $Z = \{supp, cons, lift\}$, reflecting the quality of a particular connection. The quality can also be expressed with a linear combination of the measures. The similarity between two rules r_i and r_j is defined as:

$$sim(r_j, r_j) = \frac{|Ante(r_i) \cap Ante(r_j)| + |Cons(r_i) \cap Cons(r_j)|}{|Ante(r_i) \cup Ante(r_j)| + |Cons(r_i) \cup Cons(r_j)|}, \tag{12}$$

where $Ante(.)$ denotes a set of antecedent attributes, and $Cons(.)$ a set of consequent ones. However, the $sim(r_i, r_j) \in [0, 1]$, where the value 0 means that the rules are not similar, and 1 that the rules are absolutely similar. The similarities are then combined into an adjacency matrix Adj, defined as follows:

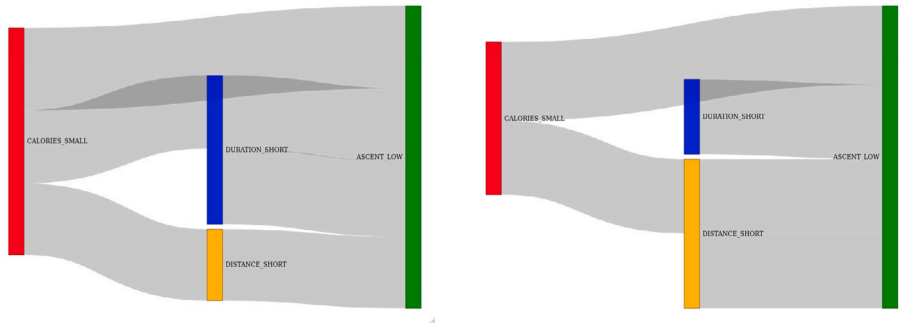
$$\text{Adj} = \begin{bmatrix} a_{1,1} & \ldots & a_{1,M} \\ & \ldots & \\ a_{\tilde{M},1} & \ldots & a_{M,M\cdot} \end{bmatrix}, \tag{13}$$

The problem of searching for the most similar set of association rules R is defined as a Knapsack 0/1 problem (Kellerer, Pferschy, & Pisinger, 2010).



Tag	Attribute
at1_e	class_edible
at1_p	class_poisonous
at5_f	bruises?_no
at5_t	bruises?_bruises
at6_n	odor_none
at7_f	gill-attachment_free
at8_c	gill-spacing_close
at9_b	gill-size_broad
at11_t	stalk-shape_tapering
at13_s	stalk-surface-above-ring_smooth
at14_s	stalk-surface-below-ring_smooth
at17_p	veil-type_partial
at18_w	veil-color_white
at19_o	ring-number_one
at20_p	ring-type_pendant
at22_v	population_several

Fig. 8. Metro map plot powered by R.



(a) Sankey diagram for time period 1. (b) Sankey diagram for time period 2.

Fig. 9. Sankey diagrams for time periods 1 and 2 powered by R.

The construction of the Sankey diagram visualization is divided into two steps: (1) searching for a set of the most similar association rules, and (2) visualization using Sankey diagrams. In Fister and Fister (2022a), the authors proposed a DE meta-heuristic algorithm using the Knapsack 0/1 deterministic algorithm for determining the set of the most similar rules, while the R programming language for statistical computing was applied to solve the second step.

The example of Sankey diagrams is illustrated in Figs. 9(a)–9(b) that refer to mining the sport training database obtained in more seasons (i.e., years). This database consists of training load indicators measured during an implementation of a sport training session. The visualization is divided into two parts: The first part (Fig. 9(a)) presents the results of the ARM visualization on sport training data captured during one season, while the second (Fig. 9(b)) highlights the data obtained during the next season.

In this way, two historical insights are served to a sport trainer: (1) In what proportion do the training load indicators contribute to the whole? and (2) What changes can be observed in the sense of training load indicators by athletes who have already had the main portion of training sessions during the previous seasons?

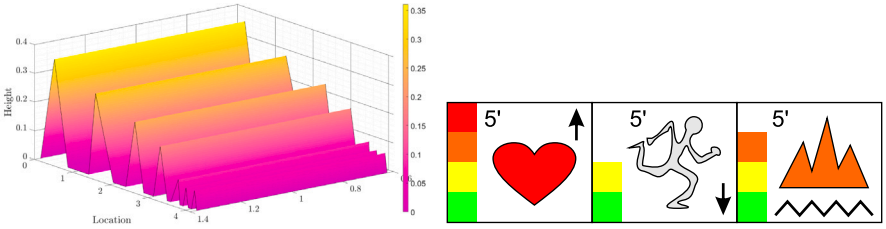
Interestingly, Hlosta, Šebek, and Zendulka (2013) proposed a visualization of evolving association rules using graphs, where the nodes of the graphs represent items and edges specific association rules. Thus, the graph-based diagram shows how evolving models mined using the ARM algorithms and stored into a transaction database can be filtered and visualized.

5.6. Ribbon plot

Ribbon plots are appropriate for visualizing data without self-intersections, where linearized simplification of events exposes the significant ones. Although the plot is ideal for analyzing linearized sequences, it can be applied successfully for visualizing the best association rule in NARM, where the proper boundaries need to be discovered between the numerical attributes. Thus, the attribute with the best support is compared with the other attributes in the association rule according to support and confidence. The attributes are ordered into linear sequence according to the closeness of the first attribute regarding the others.

The inspiration behind the visualization is presented by the Tour De France (TDF), i.e., the most famous cycling race in the world. Similar as in the TDF, where the best hill climbers have more chance to win the race, the attribute with the higher support also has the most decisive role in a decision-making process. Indeed, virtual hill slopes are visualized as triangles situated on a plain, where the left leg denotes an ascent and the right leg a descent of the virtual hill in a linear sequence, starting from the left to the right side. In the paper of Fister et al. (2020), the ascent of the virtual hill is proportional to the attribute's support, while the descent to the confidence of the simple association rule.

Mathematically, the best rule $X \Rightarrow Y$ consists of an antecedent $X = \{A_x = a_x\}$ and a consequent $Y = \{A_{y_1} = a_{y_1}, \ldots, A_{y_k} = a_{y_k}\}$, where the A_x denotes the best attribute according to the support, and



(a) Ribbon plot powered by Matlab. (b) Glyph-based chart (Hrovat et al., 2015).

Fig. 10. Ribbon plot and Glyph-based chart.

simple association rules $A_x \Rightarrow A_{y_j}$ for $j = 1, \ldots, k$ are ordered as:

$$conf(A_x \Rightarrow A_{y_{\pi_1}}) \geq conf(A_x \Rightarrow A_{y_{\pi_k}}), \tag{14}$$

where π_j is a permutation of the attributes belonging to the consequent. Moreover, the distances $dist_j$ between the virtual hills are also proportional to $dist_j \propto conf(A_x \Rightarrow A_{y_{\pi_j}})$.

An example of a ribbon plot is illustrated in Fig. 10(a) representing a visualization of the best association rule mined by the uARM-Solver (Fister & Fister, 2020) (i.e., the framework for NARM using the nature-inspired algorithms).

The framework was applied for mining a database consisting of transactions obtained by cycling training sessions. Thus, the best transaction is composed from seven attributes $A_1, \ldots, A_{k+1}$ ordered into the association rule:

$$A_x \Rightarrow A_{y_{\pi_1}} \wedge \cdots \wedge A_{y_{\pi_k}}.$$

Seven virtual hills can be observed as can be seen from the figure. While the first three virtual hills are of comparable height to the first one, the remainder of the hills are of lower height, and, thus, reflect the lower inter-dependence.

5.7. Glyph-based plots

Glyph-based plots are suitable for visualizing multivariate data with more than two attribute dimensions, where different data variables are presented by a set of visual channels (i.e., shape, size, color, orientation, curvature, etc.) (Borgo et al., 2013). Indeed, glyphs are devoted for depicting attributes of data that, typically, appear in collections of visualized objects. They are founded on the basics of a semiotic theory that is, in fact, the science of signs (Lagopoulos & Boklund-Lagopoulou, 2020). According to this theory, signs have emerged in three forms: icons, indices, and symbols. Icons reflect a physical correlation to the sign. The index expresses a space and time correlation to the object. In other words, they have an indirect effect on the object. A meta-physic correlation (i.e., no real correlation) exists between the symbol and the sign.

An example of glyph-based visualization for ARM was performed by Hrovat et al. (2015) that analyzed the time series data gathered from a single athlete (i.e., a cyclist) during a large time period of training (i.e., the whole season). In this study, the sequential pattern mining algorithm (Agrawal et al., 1994) was exploited, where the sequential patterns were discovered by employing the novel trend interestingness measure for mining sequential patterns. Thus, a time-series sequences $ts = \langle ts_1, \ldots, ts_m \rangle$ were discovered from a transaction database consisting of sport training performed by a single athlete.

Two trend interestingness measures are defined in the study as follows: (1) the duration trend $\overrightarrow{dur}(ts)$, and (2) the daily trend $\overrightarrow{dai}(ts)$. The former discovers trends within a trend database on a monthly, while the latter on a daily basis. The trend database is constructed from the original transaction database by dividing each training session into m-time series. Then, the permutation test is performed, after which those sequential patterns are selected with a minimum p-value. Obviously, the p-value is obtained as a result of the permutation test, and serves as a trend interestingness measure.

Both trend interestingness measures are visualized using glyphs in order to depict how trends increase or decrease during a specific training period (Fig. 10(b)). Thus, two glyph symbols are used by the visualization: (1) level, and (2) variable. The level's symbol depicts the trend interestingness measure using an optical channel (i.e., color), where the intensity training load indicators are presented in different colors, depending on low, moderate, intensity, and high intensity levels. The variable's symbol addresses the geometric channels, like: the cyclist's speed (as maximum, average or standard deviation), average heart rate (as minimum, maximum, average and standard deviation), and altitude (as standard deviation). These symbols are depicted using different shapes.

6. Taxonomies of the ARM visualization

The ARM visualization methods can be classified according to many aspects. These aspects depend on the various standpoints from which they are observed. Indeed, the following questions reflect those standpoints more precisely:

- How to visualize?
- Which visualization methods to use?
- Which characteristics of association rules are essential to visualize?
- What to visualize?
- Which type of attributes to display?

In the remainder of the section, these queries are described in detail.

6.1. How to visualize?

The aspect "How to visualize?" refers to the mode of how the exploration and visualization are performed. In line with this, four different methods are distinguished, as follows (Fig. 11):

- reducing the itemset,
- visual data mining,
- a concept lattice,
- evolving association rules.

Reducing the itemset means that the exploration of association rules is performed with traditional ARM methods (e.g., Apriori, Eclat, evolutionary algorithms), after which the visualization is performed using some traditional or new age visualization methods. Visual data mining comprises those ARM visualization methods that perform the exploration and visualization phases in one step. These methods mine association rules more directionally, where mining can be performed from some concept, can use meta rules, or can be able to limit the number of occlusions. The primary pipeline of this concept can be substituted with an additional ML method that succeeds by ARM, i.e., clustering. The purpose of clustering is to discover neighborhoods within the mined rule set and, thus, to contribute to reduce it before visualization. These

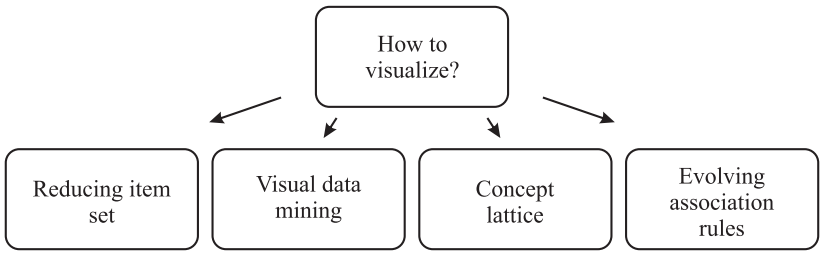


Fig. 11. How to visualize?.

Table 4
Taxonomy of the ARM visualization methods.

Method	Characteristics			Focus					Attribute			
	Nr. of interesting meas.	Rule set size	Interactive	Interesting measure	Rule length	Items	RHS+LHS	Time-series	Categorical	Numerical	Binary
Scatter plot	3	❹	●	▲					■		
Two key plot	2+	❹	●		▲				■		
Graph-based	2	❸				▲			■		
Matrix-based	1	③	●			▲			■		
Grouped matrix	2	❺	●			▲			■		
Mosaic plot	2	❶	●			▲			■		
Double Decker	2	❶	●			▲			■		
Ishikawa diagram	1	❷					▲		■	■	
Molecular representation	3	❶	●				▲		■		
Concept lattice	1	❷					▲	▲			■
Metro map	1	❷					▲	▲	■	■	
Sankey map	2	❷						▲		■	
Ribbon plot	2	❶						▲	■	■	
Glyph-based chart	1	❶							■	■	

methods are known in the study under the name "advanced reducing RS" and "advanced visual DM". The concept lattice enables displaying the structure of the association rules (i.e., attributes) beside the single rules. However, this visualization method is reserved for displaying the binary association rules only. The evolving association rules are appropriate for visualizing either warehouse data cubes stored in a multidimensional data model, or data suitable for displaying by Sankey diagrams.

6.2. Which visualization methods to use?

This aspect is focused on the question, which visualization method to use? In line with this, we can consider that the methods are divided into traditional and new age visualization methods. The former consists of charts, like scatter plot, group-based, matrix-based and mosaic plots, and their variants, like two-key, grouped-matrix and double Decker plots (see Table 4 under the column "Method"). The new age visualization methods are comprised of an Ishikawa diagram, molecular representation, a concept lattice, metro maps, Sankey diagrams, ribbon plots, and glyph-based charts.

6.3. Which characteristics of association rules are essential to visualize?

The characteristics of the ARM visualization methods refer to: (1) the number of displayed interestingness measures, (2) the rule set size, and (3) the interactivity tools. The number of displayed interestingness rules determines, how many of the interestingness measures are included into the representation for user. For instance, the scatter plot is able to display three interestingness measures, while the two-key plot actually only two, but the third measure is presented indirectly by a color. In general, the number of measures by various visualization methods are typically in the range $[1, 3]$. The rule set size determines the number of association rules to be displayed by the definite visualization method. This number is denoted in Table 4 in the column "Rule set size" in circles with numbers within them. The numbers present the powers of base 10. This means that the grouped matrix can display 10^5 association rules. The column "Interactive" shows if specific visualization method supports interactive tools (e.g., hover, zoom, pan, drill down, etc.) or not. Interestingly, although the new age visualization methods do not support interactive tools in general, they allow tuning of parameter settings that enable users some kind of interactivity.

6.4. What to visualize?

The aspect, answering to the question "What to visualize?", deals with the focus, which an ARM visualization is presenting. Actually, the ARM visualization can be focused on illustrating: (1) number of interestingness measures, (2) rule length, (3) items, (4) RHS and LHS, and (5) time series data. The first focus is devoted to displaying the number if interestingness measures. The rule length refers to the number of attributes in the visualized association rules. The item focuses on depicting the attributes of the association rules, while the RHS+LHS focus is concentrated on the structure of the more important rules. Finally, the last focus considers the time series data.

Interestingly, the concept lattice and metro maps even cover two focuses of displaying association rules, i.e., items (i.e., attributes) and their structure. On the other hand, the glyph-based visualization is dedicated for presenting the time series data.

6.5. Which type of attributes to display?

The aspect "Which type of attributes to display?" is focused on visualization based to distinguish the attribute types. In the ARM exploration/visualization, three attribute types can be identified as

Table 5
Archive of the ARM visualization papers captured in the brief review.

Nr.	Chart	How to visual.	Reference
1	Association plot	Reducing RS	Yamada, Funayama, and Yamamoto (2015)
2	Inform. landscape map	Reducing RS	Blanchard, Guillet, and Briand (2003)
3	Scatter plot	Reducing RS	Menin et al. (2021)
4	3D visualization	Reducing RS	Zheng, Guo, Fu, Li, and Li (2017)
5	Spider view	Reducing RS	Saeed, Sadaf, and Muhammad (2011)
6	Matrix-based	Reducing RS	Wong, Whitney, and Thomas (1999)
7	ROC graph	Reducing RS	Kawahara and Kawano (1999)
8	Hasse diagram	Reducing RS	Jiang, Han, and Hu (2008)
9	Scatter plot Chord diagram	Reducing RS	Liang et al. (2021)
10	3D visualization	Visual DM	Chakravarthy and Zhang (2003)
11	TS scatter plot	Visual DM	Xu, Li, Xiao, and Guo (2009)
12	3D matrix-based	Visual DM	Couturier, Hamrouni, Yahia, and Nguifo (2007)
13	3D matrix-based	Visual DM	Couturier, Dubois, Hsu, and Nguifo (2008)
14	3D histogram	Visual DM	Yahia and Nguifo (2004a)
15	3D-histogram Matrix-based	Visual DM	Yahia and Nguifo (2004b)
16	Graph-based	Visual DM	Ounifi, Amdouni, Elhoussine, and Slimane (2016)
17	3D matrix-based	Visual DM	Wang, Zhang, Chang, Ristaniemi, and Liu (2017)
18	Self organizing map	Reducing RS+	Fong, Biuk-Aghai, and Tin (2017)
19	Social network	Visual DM+	Gorecki, Slaninová, and Snášel (2011)

follows: (1) categorical, (2) numerical, and (3) binary. Interestingly, the majority of the traditional visualization methods are suitable for displaying the categorical type of attributes. Usually, displaying attributes of the numerical type is performed by these visualization methods by discretizing the numerical attributes into discrete classes. Obviously, the new age visualization methods are capable of working with the numerical and binary attributes directly as well.

7. Review of the remainder analyzed ARM visualization papers

The characteristics of the remainder analyzed papers can be summarized in the present section as follows: The majority of the papers were published for various data mining conferences. As a result, these include ideas more on the conceptual level, and, therefore, the solutions that they reveal are not robust enough for using in the everyday real-world environment. On the other hand, these ideas are not included into some recognizable ARM visualization system. However, they could be interesting for the potential readers for sure.

The ARM visualization papers, that were selected for the mentioned study, are aggregated into Table 5. The table classifies the referenced papers according to two criteria: (1) chart type and (2) how to visualize. There are a lot of chart types for ARM visualization appropriate for interacting with human visual systems (Knaflic, 2015). Let us notice that those charts, which are not mentioned previously in the study, are described shortly in the remainder of the section. On the other hand, four principles of ARM visualization can be identified according to the second criterion: reducing rule set (attribute "Reducing RS"), visual data mining (attribute "Visual DM"), advanced reducing rule set (attribute "Reducing RS+"), and advanced visual mining (attribute "Visual DM+").

Obviously, the reducing RS can be performed on many ways. For instance, Yamada et al. (2015) applied the conditional association rule analysis and the association rule analysis with user attributes for the comprehending questionnaire data. Hence, an association plot was employed for visualization. The association plot consists of items represented as circles and conditions described as rhombuses. The items are connected using arrows and represent association rules. Blanchard et al. (2003) introduced the rummaging model for filtering association rules interactively, and included into an experimental prototype called ARVis. A 3D information landscape was applied to represent the mined association rules visually. The landscape is divided into two areas: The first is dedicated to the specific rules, while the second to the general rules. In this way, the good rules are stressed, and comparison with the worse rules is made more accessible.

A similar model for exploring visualization rules was recommended by Menin et al. (2021), who represented them through RDF knowledge graphs. Hence, the traditional methods for visualizing were incorporated into the prototype ARViz (e.g., scatter plot for an overview of rules, chord diagram for a subset of rules, and association graph for itemsets). The gray correlation rule visualization algorithm was advised by Zheng et al. (2017) that is suitable for considering the influence of the association rules on the visualization. Thus, the selected association rules are illustrated using the proprietary 3D visualization, where the X-axis shows the association rules, the Y-axis their support and confidence, and the Z-axis represents the term of the rule. Saeed et al. (2011) mined a collection of documents consisting of metadata with the Apriori algorithm, and selected an association rule set for visualization according to the calculated correlation between documents. The mining results are visualized using a spider view that is typically used for displaying the multivariate data, where three or more variables (i.e., correlated weights) are represented on axes starting from the same point (i.e., base association rule).

A visualization of association rules for text mining was proposed by Wong et al. (1999), who represented the *many-to-one* association rules using 3D matrix-based chart suitable for depicting the *rule-to-item* relationship, Kawahara and Kawano (1999) performed the ARM on text obtained from own web search engine, and used a Receive Operating Characteristic (ROC) graph to evaluate the performance and characteristics of the mined rules. The ROC maps were then employed for visualization of the results represented in the ROC graph. Boolean association rules were visualized by Jiang et al. (2008) using the hierarchical structure for all of them and depicted in a Hasse diagram.

Application of visual DM in agriculture was employed by Liang et al. (2021), where authors discovered the associations between different antibiotics of essential bacteria (i.e., multidrug resistance), with which cows in large-scale farms in China are infected. The association rules were mined using a weighted Apriori ARM algorithm, while the mined rule set was reduced using a similarity comparison method based on Euclidean distance. The results were then visualized using scatter plots and chord diagrams. A chord diagram is suitable for exposing interrelationships between data in a matrix. Indeed, the diagram is based on the graph, where each node (also item) is represented as a fragment along the circle's circumference. At the same time, arcs show it flows with thicknesses proportional to the significance of the flow (Holten, 2006).

Visual data mining can be performed in various ways, as found in our study: Chakravarthy and Zhang (2003) proposed a relational SQL query language, with which a user can select the suitable association rule set for visualization interactively from the collection of association rules stored in tables. The proprietary 3D visualization was developed for displaying the set of selected mined rules, where the support and confidence measures are also presented. The correlation visualization algorithm was proposed for mining the alarm association rules by Xu et al. (2009), where the time series data were captured using multiple scatter plots.

Couturier et al. (2007) recommended the integrated framework for association rule extraction and visualization in one step, which integrated 3D bar visualization method of association rule visualization. In this study, they detected that their approach suffers from an overlapping between several objects in the 3D visualization (i.e., an occlusion). Consequently, the occlusion optimization was proposed by Couturier et al. (2008) that tried to considerable reduce the number of these phenomena. Contextual exploration of an association rule set was developed by Yahia and Nguifo (2004a, 2004b), where the additional knowledge needed for visualization was constructed using the fuzzy meta-rules. Both studies depicted the selected association rules in a 3D histogram-based visualization that presented the antecedent, consequent, and corresponding support and confidence measures. Ounifi et al. (2016) solved the problem of extraction and visualization by a 3-dimensional visualization engine, while Wang et al. (2017) introduced a 3-dimensional matrix-based visualization system, where the basic matrix-based approach was extended by rule-to-items mapping.

Fong et al. (2017) supplemented their ARM pipeline based on the Apriori ARM algorithm with clustering (reducing RS+). The clustered data were then visualized by a Self Organizing Map (SOM) ML method. The SOM is an unsupervised neural network model that can be applied for data clustering and visualization (Shieh & Liao, 2012). Gorecki et al. (2011) tackled the problem of visualizing similarities using synthetic social networks. The authors used clustering and association rule mining methods (i.e., visual DM+) to discover and represent potentially exciting similarities in the data to model a synthetic social network. Results were visualized using a graph-based approach.

8. ARM visualization systems

The section aims to compile a list of specialized ARM visualization systems and software packages for any of the ARM visualization methods. Obviously, this does not present the other visualization libraries, from which we can develop some methods (e.g., **matplotlib** in Python, or **ggplot2** in R). On the other hand, the review does not discuss copyright systems, like PARAS/FIRE (Mukherji et al., 2013) using the parameter space-based ARM methodology (PARAS) and the framework for interactive rule exploration (FIRE), that offers the integrated interactive framework for association rule mining and visualization. In line with this, the study focused on presenting this collection of graphics system that are more commonly used today in the ARM community. The collection of systems is illustrated in Table 6.

As can be seen from the table, the **arulesViz** graphics system is the most complete, due to covering the majority of the visualization methods dealt with in this review paper. This is an extensive toolbox in the R-extension package (Hahsler et al., 2011), and works in two phases: (1) exploration using known ARM methods to which tools for reducing the huge number of association rules are applied (e.g., filtering, zooming and rearranging), and (2) visualization of results. The current version of the software supports the following visualization methods (i.e., graphics): scatter plots, network plots, matrix-based, graph-based, mosaic plots and parallel coordinate plots.

The other libraries are just a smaller drop in the ocean and, typically, they solve only limited ARM exploration/visualization approaches. For example, while the **NiaARM** is focused at this moment on only one visualization method (i.e., ribbon plot), the **PyARMviz** graphics system tends to be what is arulesViz for R, but in Python. Unfortunately, the development of this graphics software has probably stalled since the last commit was done almost three years ago. On the other hand, the development of the NiaARM is not finished yet, due to the unfinished inclusion of the new ideas in ARM visualization (e.g., metro maps, Sankey diagram, etc.) that should shortly widen the usability of the graphics system.

Table 6
List of the ARM graphics systems.

R packages	
arulesViz https://cran.r-project.org/web/packages/arulesViz/index.html	1.1 Probably the only state-of-the-art tool that supports many visualization methods up to this date 1.2 Includes also interactive tools
Python packages	
pycaret https://github.com/pycaret/pycaret	2.1 Basically low-code machine learning library in Python 2.2 Association rule mining is a part of this library 2.3 Library supports 2D and 3D plots of association rules
NiaARM (https://github.com/firefly-cpp/NiaARM)	3.1 Minor module devoted for visualization 3.2 For now supports only ribbon plots
PyARMViz https://github.com/Mazeofthemind/PyARMViz	4.1 Python Association Rule Visualization Library that is loosely based on ArulesViz 4.2 Development probably stalled (no commits in the last 2.5 years)
C++ packages	
uARMSolver https://github.com/firefly-cpp/uARMSolver	5.1 Small part of this package is devoted to the visualization 5.2 Provides the coordinates for metro plots which can be later visualized using metro map algorithms

9. Challenges and open problems

Visualizations play a crucial role in data analysis and decision-making processes. These visualizations enable data professionals to quickly understand a data set's patterns, trends, or outliers. Data visualization also presents data to the general public or specific audiences without technical knowledge in an accessible manner. This aims to help drive informed decision-making and add enriched meaning to an otherwise tedious database (Coursera Inc., 2023).

In summary, ARM visualizations provide decision-makers with a powerful tool for understanding, analyzing, and communicating complex information. They enhance comprehension, support pattern recognition, aid exploration, and discovery, and facilitate effective communication. However, it is essential to choose appropriate visualization techniques, consider scalability, and understand the limitations associated with data quality and interpretation (Cairo, 2012).

After a deep analysis of the ARM visualization methods, the one-focused view on the visualization rule set is usually insufficient for the user. Indeed, they often apply top-down approach by analyzing and visualizing the rule set. According to this approach, the ARM visualization is performed in three levels corresponding to different visualization focuses: At the higher level (first level), an overview of association rules in the visualization rule set is illustrated using various ARM measures (most often support and confidence). The scatter plots are typically used for these purposes, where each association rule is presented as a dot in a chart. At the next level (second level), connections between selected rules are observed. Thus, the ARM visualization is focused on RHS/LHS, where the charts like matrix-based, grouped matrix, and Sankey diagrams, are more suitable. At the last level (i.e., third level), the ARM visualization focus is devoted to items, where charts,

like molecular representation, chord diagram, and graph-based, are more preferable. Interestingly, some ARM visualizations from this level support the single rule only, where the charts, like spider views, mosaic plots, double Decker, and ribbon plots, are more appropriately.

For instance, the arulesViz software package (Hahsler et al., 2011) offers a spectrum of solutions useful for visualization with traditional ARM visualization methods. In this package, the scatter plot is applied as an entry point for an analysis of how to distinguish the similarity of association rules according to support and confidence interestingness measures. Then, the matrix-based visualization can be applied, capable of organizing association rules into a matrix, where the antecedent and consequent items can be distinguished. Finally, the graph-based methods are recommended by authors, in order to get the user the broadest view of the relationships between individual items reflecting, their memberships in different association rules.

Along with the mentioned ARM visualization levels, the recommended size of the visualization rule set decreases by increasing the levels. Indeed, if the scatter plot recommends visualizing up to 1000, the visualization methods of the second level are less than 1000, and the third level visualization methods are up to 100 association rules.

In summary, the problems caused by using the traditional ARM visualization methods can be aggregated as follows (Shen et al., 2020):

- the domain knowledge is not displayed sufficiently, i.e., the rules are displayed from a single point of view,
- the visualization of background knowledge is not enough for sharing, i.e., the role and relationship of global information is lost in the context of the background knowledge,
- the use and exploration of potential knowledge hidden in non-connected attributes are reduced.

However, the new age ARM visualization methods tries to reveal the aforementioned problems. Moreover, some of these methods are even able to tell stories in mined data (e.g., metro maps), while the others are able to analyze the information from the history point of view (e.g., Sankey diagrams).

Although searching for a new age ARM visualization methods almost stopped after the rapid development of the traditional ARM visualization methods in the past, in our opinion, the future of the ARM visualization remains in the development of the new age ARM visualization methods. These methods might consolidate displaying items as well as the structure of the association rules. Additionally, these need to be independent of the attribute types.

The main advantage of the ARM visualization undoubtedly presents the interactivity of the ARM visualization methods. Interactive visualization improves the user's experience and interpretation of the results. Although several popular implementations of the traditional ARM visualization methods (e.g., arulesViz R-package by Hahsler and Karpienko (2017), and InterVisAR by Cheng, Sha, and Wang (2016)) already offer some interactive tools (e.g., hover, zoom, pan, drill down, inspect, brush), these tools are usually missing in some of the observed new age ARM visualization methods (e.g., metro maps, Sankey diagram, ribbon plot, glyph-based plot). Obviously, these tools need to be included into their visualization in the future.

10. Conclusions

Data mining methods today suffer from a lot of comprehension of the mass results they produce. In line with this, a new domain of AI, the so-called XAI, has emerged that searches for methods which will be suitable to present these results clearly to the user. The visualization methods are one of the useful tools for helping users understand the results of different data mining methods better.

The present study has revised the most important visualization methods associated with ARM. Consequently, the most important ARM visualization methods, published in research papers, have been identified, analyzed, and classified. Thus, the research papers were selected from the Google Scholar database, the most complete database to which also the ACM Digital Library and IEEE Xplore were added due to improve the search results reliability. The ARM visualization methods are divided into traditional and new age methods. Moreover, they have been classified according to the characteristics of the displayed association rules, the focus of visualization, and the types of attributes.

The potential reader of this work will be able to get deeper overview of the ARM exploration/visualization process. Furthermore, it encourages readers to open new avenues of potential research. According to the research paper review, there is a huge opportunity to use the knowledge, especially in biological/medical sciences.

CRediT authorship contribution statement

Iztok Fister Jr.: Conceptualization, Project administration, Writing – original draft. **Iztok Fister:** Investigation, Resources, Validation, Writing – review & editing. **Dušan Fister:** Data curation, Visualization. **Vili Podgorelec:** Supervision, Resources, Validation, Funding acquisition, Writing – review & editing. **Sancho Salcedo-Sanz:** Supervision, Resources, Validation, Funding acquisition, Writing – review & editing.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Dr. Sancho Salcedo-Sanz reports financial support was provided by Spanish Ministry of Science and Innovation (MICINN). Dr. Iztok Fister Jr., Dr. Iztok Fister, Dr. Dušan Fister, Dr. Vili Podgorelec report financial support was provided by Slovenian Research and Innovation Agency (ARIS). We want to declare that Iztok Fister Jr. is the associate editor of Expert Systems with Applications journal. We have no other conflicts of interest to disclose.

Data availability

No data was used for the research described in the article

Acknowledgments

This research has been supported partially by the project PID2020-115454GB-C21 of the Spanish Ministry of Science and Innovation (MICINN). The authors acknowledge the financial support from the Slovenian Research and Innovation Agency (ARIS) (Research Core Funding No. P2-0057 & P2-0042).

References

Agrawal, R., Srikant, R., et al. (1994). Fast algorithms for mining association rules. In *Proc. 20th int. conf. very large data bases, VLDB, Vol. 1215* (pp. 487–499).

Altay, E. V., & Alatas, B. (2019). Performance analysis of multi-objective artificial intelligence optimization algorithms in numerical association rule mining. *Journal of Ambient Intelligence and Humanized Computing*, 1–21.

Arrieta, A. B., Díaz-Rodríguez, N., Del Ser, J., Bennetot, A., Tabik, S., Barbado, A., et al. (2020). Explainable Artificial Intelligence (XAI): Concepts, taxonomies, opportunities and challenges toward responsible AI. *Information Fusion*, *58*, 82–115.

Barredo Arrieta, A., Díaz-Rodríguez, N., Del Ser, J., Bennetot, A., Tabik, S., Barbado, A., et al. (2019). Explainable artificial intelligence (xai): Concepts, taxonomies, opportunities and challenges toward responsible ai. arXiv, arXiv–1910.

Bayardo, R. J., & Agrawal, R. (1999). Mining the most interesting rules. In *Proceedings of the fifth ACM SIGKDD international conference on knowledge discovery and data mining* KDD '99, (pp. 145–154). New York, NY, USA: Association for Computing Machinery, http://dx.doi.org/10.1145/312129.312219.

Bennetot, A., Donadello, I., El Qadi, A., Dragoni, M., Frossard, T., Wagner, B., et al. A practical guide on explainable ai techniques applied on biomedical use case applications. Available at SSRN 4229624.

Blanchard, J., Guillet, F., & Briand, H. (2003). A user-driven and quality-oriented visualization for mining association rules. In *Third IEEE international conference on data mining* (pp. 493–496). IEEE.

Borgelt, C. (2005). An implementation of the FP-growth algorithm. In *Proceedings of the 1st international workshop on open source data mining: Frequent pattern mining implementations* (pp. 1–5).

Borgo, R., Kehrer, J., Chung, D. H., Maguire, E., Laramee, R. S., Hauser, H., et al. (2013). Glyph-based visualization: Foundations, design guidelines, techniques and applications. *Eurographics State of the Art Reports*, 39–63, URL: https://www.cg.tuwien.ac.at/research/publications/2013/borgo-2013-gly/. http://diglib.eg.org/EG/DL/conf/EG2013/stars/039-063.pdf.

Borrego-Díaz, J., & Galán-Páez, J. (2022). Explainable artificial intelligence in data science. *Minds and Machines*, *32*(3), 485–531. http://dx.doi.org/10.1007/s11023-022-09603-z.

Buono, P., & Costabile, M. F. (2005). Visualizing association rules in a framework for visual data mining. In M. Hemmje, C. Niederée, & T. Risse (Eds.), *From integrated publication and information systems to information and knowledge environments: Essays dedicated to Erich J. Neuhold on the occasion of his 65th birthday* (pp. 221–231). Berlin, Heidelberg: Springer Berlin Heidelberg, http://dx.doi.org/10.1007/978-3-540-31842-2_22.

Cairo, A. (2012). *The functional art: An introduction to information graphics and visualization* (1st ed.). USA: New Riders Publishing.

Chakravarthy, S., & Zhang, H. (2003). Visualization of association rules over relational DBMSs. In *Proceedings of the 2003 ACM symposium on applied computing* (pp. 922–926).

Cheng, C.-W., Sha, Y., & Wang, M. D. (2016). Intervisar: An interactive visualization for association rule search. In *Proceedings of the 7th ACM international conference on bioinformatics, computational biology, and health informatics* (pp. 175–184).

Coursera Inc. (2023). Data visualization: Definition, benefits, and examples. URL: https://www.coursera.org/articles/data-visualization.

Couturier, O., Dubois, V., Hsu, T., & Nguifo, E. M. (2008). Optimizing occlusion appearances in 3D association rules visualization. In *2008 4th international IEEE conference intelligent systems, Vol. 2* (pp. 15–42). IEEE.

Couturier, O., Hamrouni, T., Yahia, S. B., & Nguifo, E. M. (2007). A scalable association rule visualization towards displaying large amounts of knowledge. In *2007 11th international conference information visualization (IV'07)* (pp. 657–663). IEEE.

Ebbing, D., & Gammon, S. (2016). *General chemistry*. Cengage Learning, URL: https://books.google.nl/books?id=BnccCgAAQBAJ.

Ertek, G., & Demiriz, A. (2006). A framework for visualizing association mining results. In A. Levi, E. Savaş, H. Yenigün, S. Balcısoy, & Y. Saygın (Eds.), *Computer and information sciences – ISCIS 2006* (pp. 593–602). Berlin, Heidelberg: Springer Berlin Heidelberg.

Fister, I., & Fister, I. (2022a). Association rules over time. In M. Khosravy, N. Gupta, & N. Patel (Eds.), *Frontiers in nature-inspired industrial optimization* (pp. 1–16). Singapore: Springer Singapore, http://dx.doi.org/10.1007/978-981-16-3128-3_1.

Fister, I., & Fister, I. (2022b). Information cartography in association rule mining. *IEEE Transactions on Emerging Topics in Computational Intelligence*, *6*(3), 660–676. http://dx.doi.org/10.1109/TETCI.2021.3074919.

Fister, I., Fister, D., Iglesias, A., Galvez, A., Osaba, E., Del Ser, J., et al. (2020). Visualization of numerical association rules by hill slopes. In C. Analide, P. Novais, D. Camacho, H. Yin (Eds.), *Intelligent data engineering and automated learning – IDEAL 2020* (pp. 101–111). Cham: Springer International Publishing.

Fister, I., & Fister, I., Jr. (2020). uARMSolver: A framework for association rule mining. CoRR, abs/2010.10884. URL: https://arxiv.org/abs/2010.10884.

Fister, I., Fister, I., Jr., Novak, D., & Verber, D. (2022). Data squashing as preprocessing in association rule mining. In *2022 IEEE symposium series on computational intelligence* SSCI, (pp. 1720–1725). IEEE.

Fong, S., Biuk-Aghai, R. P., & Tin, S. (2017). Visual clustering-based apriori ARM methodology for obtaining quality association rules. In *Proceedings of the 10th international symposium on visual information communication and interaction* (pp. 69–70).

Freitas, A. A. (1998). On objective measures of rule surprisingness. In J. M. Żytkow, & M. Quafafou (Eds.), *Principles of data mining and knowledge discovery* (pp. 1–9). Berlin, Heidelberg: Springer Berlin Heidelberg.

Gorecki, J., Slaninová, K., & Snášel, V. (2011). Visual investigation of similarities in global terrorism database by means of synthetic social networks. In *2011 international conference on computational aspects of social networks (CASoN)* (pp. 255–260). IEEE.

Gupta, N., Mangal, N., Tiwari, K., & Mitra, P. (2006). Mining quantitative association rules in protein sequences. In G. J. Williams, & S. J. Simoff (Eds.), *Data mining: Theory, methodology, techniques, and applications* (pp. 273–281). Berlin, Heidelberg: Springer Berlin Heidelberg, http://dx.doi.org/10.1007/11677437_21.

Hahsler, M., Chelluboina, S., Hornik, K., & Buchta, C. (2011). The arules R-package ecosystem: Analyzing interesting patterns from large transaction data sets. *Journal of Machine Learning Research*, *12*(57), 2021–2025, URL: http://jmlr.org/papers/v12/hahsler11a.html.

Hahsler, M., & Karpienko, R. (2017). Visualizing association rules in hierarchical groups. *Journal of Business Economics*, *87*(3), 317–335. http://dx.doi.org/10.1007/s11573-016-0822-8.

Han, J., Pei, J., & Yin, Y. (2000). Mining frequent patterns without candidate generation. In *ACM sigmod record, Vol. 29* (pp. 1–12). ACM.

Hartigan, J. A., & Kleiner, B. (1984). A mosaic of television ratings. *JSTOR: The American Statistician*, *38*(1), 32–35. http://dx.doi.org/10.2307/2683556.

Hartigan, J. A., & Wong, M. A. (1979). A k-means clustering algorithm. *JSTOR: Applied Statistics*, *28*(1), 100–108.

Hian-Huat Ong, K., leong Ong, K., keong Ng, W., & peng Lim, E. (2002). Crystalclear: active visualization of association rules. In *International workshop on active mining, AM2002*.

Hlosta, M., Šebek, M., & Zendulka, J. (2013). Approach to visualisation of evolving association rule models. In *2013 second international conference on informatics & applications* ICIA, (pp. 47–52). IEEE.

Hofmann, H. (2000). Exploring categorical data: interactive mosaic plots. *Metrika*, *51*(1), 11–26. http://dx.doi.org/10.1007/s001840000041.

Hofmann, H. (2008). Mosaic plots and their variants. In *Handbook of data visualization* (pp. 617–642). Berlin, Heidelberg: Springer Berlin Heidelberg, http://dx.doi.org/10.1007/978-3-540-33037-0_24.

Hofmann, H., Siebes, A. P. J. M., & Wilhelm, A. F. X. (2000). Visualizing association rules with interactive mosaic plots. In *Proceedings of the sixth ACM SIGKDD international conference on knowledge discovery and data mining* KDD '00, (pp. 227–235). New York, NY, USA: Association for Computing Machinery, http://dx.doi.org/10.1145/347090.347133.

Hofmann, H., & Wilhelm, A. F. X. (2001). Visual comparison of association rules. *Computational Statistics*, *16*(3), 399–415. http://dx.doi.org/10.1007/s001800100075.

Holten, D. (2006). Hierarchical edge bundles: Visualization of adjacency relations in hierarchical data. *IEEE Transactions on Visualization and Computer Graphics*, *12*(5), 741–748. http://dx.doi.org/10.1109/TVCG.2006.147.

Hrovat, G., Fister, I., Jr., Yermak, K., Stiglic, G., & Fister, I. (2015). Interestingness measure for mining sequential patterns in sports. *Journal of Intelligent & Fuzzy Systems*, *29*(5), 1981–1994. http://dx.doi.org/10.3233/IFS-151676.

Jiang, B., Han, C., & Hu, X. (2008). A finite ranked poset and its application in visualization of association rules. In *2008 IEEE international conference on granular computing* (pp. 322–325). IEEE.

Kawahara, M., & Kawano, H. (1999). Performance evaluation and visualization of association rules using receiver operating characteristic graph. In *Proceedings 1999 international symposium on database applications in non-traditional environments (DANTE'99)(Cat. No. PR00496)* (pp. 74–83). IEEE.

Kellerer, H., Pferschy, U., & Pisinger, D. (2010). *Knapsack problems*. Springer Berlin Heidelberg, URL: https://books.google.si/books?id=Mi5bcgAACAAJ.

Kitchenham, B., Charters, S., et al. (2007). Guidelines for performing systematic literature reviews in software engineering version 2.3. *Engineering*, *45*(4ve), 1051.

Klemettinen, M., Mannila, H., Ronkainen, P., Toivonen, H., & Verkamo, A. I. (1994). Finding interesting rules from large sets of discovered association rules. In *Proceedings of the third international conference on information and knowledge management* CIKM '94, (pp. 401–407). New York, NY, USA: Association for Computing Machinery, http://dx.doi.org/10.1145/191246.191314.

Knaflic, C. (2015). *Storytelling with data: A data visualization guide for business professionals*. Wiley, URL: https://books.google.si/books?id=retRCgAAQBAJ.

Kumar, A. (2022). What is explainable AI? Concepts & examples. URL: https://vitalflux.com/what-is-explainable-ai-concepts-examples/.

Lagopoulos, A., & Boklund-Lagopoulou, K. (2020). *Semiotics, communication and cognition [SCC], Theory and methodology of semiotics: The tradition of ferdinand de saussure*. De Gruyter, URL: https://books.google.si/books?id=LXcGEAAAQBAJ.

Liang, B., Li, X., Zhang, Z., Wu, C., Liu, X., & Zheng, Y. (2021). Multidrug resistance analysis method for pathogens of cow mastitis based on weighted-association rule mining and similarity comparison. *Computers and Electronics in Agriculture*, *190*, Article 106411.

Malerba, D., Lisi, F. A., Appice, A., & Sblendorio, F. (2002). Mining spatial association rules in census data: a relational approach. In *Proceedings of the ECML/PKDD, Vol. 2* (pp. 80–93).

Menin, A., Cadorel, L., Tettamanzi, A., Giboin, A., Gandon, F., & Winckler, M. (2021). ARViz: Interactive visualization of association rules for RDF data exploration. In *2021 25th international conference information visualisation* IV, (pp. 13–20). IEEE.

Mukherji, A., Lin, X., Whitehouse, J., Botaish, C. R., Rundensteiner, E. A., & Ward, M. O. (2013). Fire: interactive visual support for parameter space-driven rule mining. In *Proceedings of the 22nd ACM international conference on information & knowledge management* (pp. 2447–2452).

Nisbet, R., Miner, G., & Yale, K. (2018). Advanced algorithms for data mining. In R. Nisbet, G. Miner, & K. Yale (Eds.), *Handbook of statistical analysis and data mining applications (Second edition)* (2nd ed.). (pp. 149–167). Boston: Academic Press, http://dx.doi.org/10.1016/B978-0-12-416632-5.00008-6, URL: https://www.sciencedirect.com/science/article/pii/B9780124166325000086.

Ounifi, M. S., Amdouni, H., Elhoussine, R. B., & Slimane, H. (2016). New 3D visualization and validation tool for displaying association rules and their associated classifiers. In *2016 20th international conference information visualisation* IV, (pp. 152–158). IEEE.

Rainsford, C. P., & Roddick, J. F. (2000). Temporal interval logic in data mining. In *Proceedings of the 6th pacific rim international conference on artificial intelligence* PRICAI '00, (p. 798). Berlin, Heidelberg: Springer-Verlag.

Saeed, Z., Sadaf, A., & Muhammad, S. (2011). Activity-based correlation of personal documents and their visualization using association rule mining. In *2011 7th international conference on emerging technologies* (pp. 1–7). IEEE.

Said, Z. B., Guillet, F., Richard, P., Picarougne, F., & Blanchard, J. (2013). Visualisation of association rules based on a molecular representation. In *2013 17th international conference on information visualisation* (pp. 577–581). IEEE.

Shahaf, D., Guestrin, C., & Horvitz, E. (2012). Trains of thought: Generating information maps. In *Proceedings of the 21st international conference on world wide web* WWW '12, (pp. 899–908). New York, NY, USA: Association for Computing Machinery, http://dx.doi.org/10.1145/2187836.2187957.

Shahaf, D., Guestrin, C., Horvitz, E., & Leskovec, J. (2015). A metro map can tell a story, as well as provide good directions. *Communications of the ACM*, *58*(11), 62–73.

Shen, X., Bao, L., & Zhang, L. (2020). Research on visualization algorithm of association rules based on concept lattice. In *Proceedings of the 2020 international conference on cyberspace innovation of advanced technologies* (pp. 22–27).

Shieh, S.-L., & Liao, I.-E. (2012). A new approach for data clustering and visualization using self-organizing maps. *Expert Systems with Applications*, *39*(15), 11924–11933. http://dx.doi.org/10.1016/j.eswa.2012.02.181, URL: https://www.sciencedirect.com/science/article/pii/S0957417412004447.

Soysal, Ö. M. (2015). Association rule mining with mostly associated sequential patterns. *Expert Systems with Applications*, *42*(5), 2582–2592.

Tague, N. (2005). *The quality toolbox* (2nd ed.). ASQ Quality Press, URL: https://books.google.si/books?id=G3c6S0mzLQgC.

Telikani, A., Gandomi, A. H., & Shahbahrami, A. (2020). A survey of evolutionary computation for association rule mining. *Information Sciences*.

Truong, T. C., & Tran, A. N. (2010). Structure of set of association rules based on concept lattice. In N. T. Nguyen, R. Katarzyniak, & S.-M. Chen (Eds.), *Advances in intelligent information and database systems* (pp. 217–227). Berlin, Heidelberg: Springer Berlin Heidelberg, http://dx.doi.org/10.1007/978-3-642-12090-9_19.

Tsurinov, P., Shpynov, O., Lukashina, N., Likholetova, D., & Artyomov, M. (2021). FARM: hierarchical association rule mining and visualization method. In *Proceedings of the 12th ACM conference on bioinformatics, computational biology, and health informatics* (p. 1).

UC Irvine ML Repository (1987). UCI machine learning repository. URL: https://archive-beta.ics.uci.edu/.

Unwin, A., Hofmann, H., & Bernt, K. (2001). The twokey plot for multiple association rules control. In L. De Raedt, & A. Siebes (Eds.), *Principles of data mining and knowledge discovery* (pp. 472–483). Berlin, Heidelberg: Springer Berlin Heidelberg.

Wang, B., Zhang, T., Chang, Z., Ristaniemi, T., & Liu, G. (2017). 3D matrix-based visualization system of association rules. In *2017 IEEE international conference on computer and information technology* CIT, (pp. 357–362). IEEE.

Wong, P. C., Whitney, P., & Thomas, J. (1999). Visualizing association rules for text mining. In *Proceedings 1999 IEEE symposium on information visualization (InfoVis' 99)* (pp. 120–123). IEEE.

Wu, X., Kumar, V., Quinlan, J. R., Ghosh, J., Yang, Q., Motoda, H., et al. (2008). Top 10 algorithms in data mining. *Knowledge and Information Systems*, *14*(1), 1–37.

Xu, Q., Li, C., Xiao, B., & Guo, J. (2009). A visualization algorithm for alarm association mining. In *2009 IEEE international conference on network infrastructure and digital content* (pp. 326–330). IEEE.

Xu, W., Zhao, Q., Zhan, Y., Wang, B., & Hu, Y. (2022). Privacy-preserving association rule mining based on electronic medical system. *Wireless Networks*, *28*(1), 303–320. http://dx.doi.org/10.1007/s11276-021-02846-1, PMCID: PMC8720560.

Yahia, S. B., & Nguifo, E. M. (2004a). Contextual generic association rules visualization using hierarchical fuzzy meta-rules. In *2004 IEEE international conference on fuzzy systems (IEEE Cat. No. 04CH37542), Vol. 1* (pp. 227–232). IEEE.

Yahia, S. B., & Nguifo, E. M. (2004b). Emulating a cooperative behavior in a generic association rule visualization tool. In *16th IEEE international conference on tools with artificial intelligence* (pp. 148–155). IEEE.

Yamada, S., Funayama, T., & Yamamoto, Y. (2015). Visualization of relations of stores by using association rule mining. In *2015 13th international conference on ICT and knowledge engineering (ICT & knowledge engineering 2015)* (pp. 11–14). IEEE.

Yang, L. (2005). Pruning and visualizing generalized association rules in parallel coordinates. *IEEE Transactions on Knowledge and Data Engineering*, *17*(1), 60–70.

Zaki, M. J. (2000). Scalable algorithms for association mining. *IEEE Transactions on Knowledge and Data Engineering*, *12*(3), 372–390.

Zheng, Z.-y., Guo, J.-z., Fu, Y., Li, L., & Li, D. (2017). Visualization of association rules based on a gray correlation analysis model. In *2017 13th international conference on natural computation, fuzzy systems and knowledge discovery (ICNC-FSKD)* (pp. 638–643). IEEE.